COMPUTABILITY WITH PARTIAL INFORMATION

Introduction

Computability theory is one of the main branches of mathematical logic. It originates in the 1930s with the study of the first formal computational models such as Turing machines, Church's λ-calculus, Post canonical systems and others. The basic properties of the computable functions are established mainly in the works of Kleene. Post suggests thereafter in the 1940s a reducibility of sets of natural numbers, based on the notion of relative Turing computability and called Turing reducibility.

The study of Turing reducibility leads to the notion of a Turing degree, a class of Turing equivalent sets, and to the structure of the Turing degrees, an upper semi-lattice with least element.

The Turing degrees have been studied by a number of mathematicians, motivated by the wish to understand the world of sets of natural numbers, structured according to their information content. It emerges that the theory of the Turing degrees is mathematically non-trivial, rich in ideas and results, which determines its further development.

Besides Turing reducibility, other reducibilities such as m-reducibility, truth table reducibility and weak truth table reducibility appear. Each of these leads in a natural way to a corresponding degree structure.

Among these reducibilities of special interest is enumeration reducibility, defined by Friedberg and Rogers [17]. A set A is enumeration reducible to a set B if there is an effective way to obtain an enumeration of A given any enumeration of B. This reducibility gives the most general way to compare the positive information content of two sets. Enumeration reducibility is based on nondeterministic Turing computability. The enumeration operators give the semantics of the type free λ-calculus in graph models, suggested by Plotkin in 1972. The interest in enumeration reducibility is also supported by the the fact that the structure of the enumeration degrees contains the structure of the Turing degrees without being elementary equivalent to it. Contemporary results in the theory of enumeration degrees show that the enumeration degrees are useful for the study of the structure of Turing degrees.

1. The scientific research in computability theory in Bulgaria

The Bulgarian school in Computability theory is founded by Dimitar Skordev. In the 1970s Skordev initiates the development of algebraic recursion theory, investigated in his monographs [1, 39]. The study of algebraic recursion theory is continued by his students I. Zashev and L. Ivanov

In the 1980s Skordev directs two other students, A. Dichev and I. Soskov, to the study of models of computation in abstract structures. The main problem that he suggests is to clarify the connections between two basic approaches towards abstract computability: the internal approach, based on specific models of computation, and the external approach, which defines the computable functions through invariance relative to all enumerations of a structure. Later this research group is joined by A. Soskova and S. Nikolova. Problems from degree theory arise in a natural way in the context of this research. The first results in this area are obtained by A. Dichev [14, 11, 13, 12, 15].

Towards the end of the last century I. Soskov starts his work in degree theory. His main motivation comes from the ideological connections between one of the models of abstract computability, called search computability, and enumeration reducibility. In his works [44, 45, 50, 49, 52] Soskov and his students develop the theory of regular enumerations and apply it to the enumeration degrees, obtaining a series of new results, mainly in relation to the enumeration jump.

The connection between enumeration degrees and abstract models of computability is investigated in the article [46]. There a new direction in the field of computable model theory is given, namely the study of the properties of the spectra of structures as sets of enumeration degrees. This theme is further developed also by A. Soskova [59, 53, 54, 57, 55, 56, 61].

In 2006 Soskov initiates the study of uniform reducibility between sequences of sets and the induced structure of the ω-degrees. With his student H. Ganchev they obtain many results, providing substantial proof that the structure of the ω-degrees is a natural extension of the structure of the enumeration degrees, where the jump operator has interesting properties and where new degrees appear, which turn out to be extremely useful for the characterization of certain classes of enumeration degrees [47, 48, 51, 19, 18].

2. The scientific research in computability theory in Europe

The strongest centers in the area of computability theory and degrees of unsolvability are in Leeds (Cuper), Sienna (Sorbi), Heidelberg (Ambos-Spies) Kazan (Arslanov, Kalimullin), Novosibirsk (Ershov, Goncharov) and Sofia.

In the schools abroad the researchers mainly investigate problems connected to Turing and enumeration degrees.

The structure of the ω-enumeration degrees is studied mainly in Sofia. Sofia school has published several papers in international journals and delivered several talks on international conferences which shows that the ω-enumeration degrees are well accepted.

There are several journals in Europe as "Archive of Mathematical Logic" and "Mathematical Logic Quarterly", "Journal of Logic and Computation", "Mathematical structures in Computer Science", etc., which publish papers in Computability Theory.

Since 2005 the Computability in Europe network has been very active organizing the series of conferences under the same name. After the very successful conferences in Amsterdam, Swansea, Sienna and Athens, the next will be in 2009 in Heidelberg, in 2010 in the Azores, and in 2011 in Sofia.

3. Description of the university or the scientific group — beneficent

Sofia University "St. Kliment Ohridski" is the oldest and the leading research and teaching university of Bulgaria. The university offers Bachelor's, Master's and doctoral degrees in natural and social sciences and humanities.

The structure of the university includes 16 Faculties, 3 Department and numerous scientific centers and laboratories. During the academic year 07/08 the university is training 25 060 students distributed in the following streams: 18 235 in the bachelor degree programmes, 4 656 in the master degree programmes, 829 in the doctoral degree programmes and 1 340 foreign students. The academic staff numbers 1 800 lecturers, among them 600 full professors and associate professors. Sofia University is one of the best scientific centers in Bulgaria. The most important characteristic of the scientific research in Sofia university is the connection with the education.

The Faculty of Mathematics and Informatics (FMI) (www.fmi.uni-sofia.bg) has its origins in the Department of Physics and Mathematics and was established in 1889 as a subdivision of the first Bulgarian Higher School. In 1904 the Higher School became Sofia University "St. Kliment Ohridski" with a separate Faculty of Physics and Mathematics. Since 1963 the Faculty of Mathematics exists as a separate faculty. In 1968 it takes the name of Faculty of Mathematics and Mechanics and from 1986 it is named Faculty of Mathematics and Informatics. FMI is the Sofia University teaching and research center for Pure Mathematics, Applied Mathematics, Education in mathematics and informatics, Computer Science, Information technologies and Software Engineering. There are over 2 200 undergraduate and graduate students studying in the faculty at the moment. Presently with more than 150 researchers FMI is among the largest departments of Sofia University. The full time employees are about 70 Professors and Associate Professors and over 80 Assistant Professors. Many of them have taught at renowned universities in Europe, USA, Canada, and and have participated in different international scientific forums and symposia. This accounts for the high quality of the teaching process at the FMI proof of which is the performance of its students at international competitions in mathematics and informatics. FMI is subdivided into 15 chairs and additional research groups (laboratories etc.). At present FMI has 36 PhD students. During the last 10 years more than 300 Bulgarian and foreign PhD students in all fields of fundamental and applied mathematics and Computer Science successfully finished their thesis. The research in mathematics and Computer Science is very intensive where the PhD students are involved

The faculty is involved in over 30 projects – part of the EU Framework Programmes. In this relation, FMI has established good connections with other European research centers and has a good reputation among them.

The Faculty has well developed research infrastructure. FMI has more than 10 modern computer labs and two big libraries. The faculty libraries dispose of about 8 000 volumes, including a rich collection of the oldest Balkan mathematical literature. The faculty offices and laboratories are situated in the university buildings on 5 James Bourchier blvd. and Block 2 (on Tsarigradsko shose, blvd.). FMI disposes with a large number of computers to which there is a regulated access. All FMI computers are part of the faculty local network. In this way, the staff and students of the faculty have free access to Internet, as licenced software and freeware. An office in the old building of the Faculty of Physics will be supplied for the administrative purposes of this project.

4. Description of the work program

The goal of this project is to continue the investigation of the structures $\mathcal{D}_\omega$ of the ω-enumeration degrees and $\mathcal{D}_e$ of the enumeration degrees, and in particular to clarify the connections between the two structures. The work will lay emphasis on problems connected to the first order definability in both structures, to embeddings of substructures of a particular type and to the characterization of the automorphism groups of the structures.

The obtained results shall be applied to research the properties of the spectra of structures.

4.1. **Current and past scientific problems in the field.**

4.1.1. *Enumeration degrees.* Following Friedberg and Rogers [17], A is said to be enumeration reducible to B ($A \leq B$) if there exists an effective procedure for obtaining an enumeration of A from any enumeration of B. It turns out that this relation is the most general well behaved means of comparing the positive information content of sets. Indeed, Selman proved in [37] that this reducibility is a maximal transitive relation of *is* Σ^0_1 *in*. Much of the present interest in enumeration reducibility stems from its relationship with the most widely studied relation in computability theory, Turing reducibility ($\leq_T$) and the latter's degree structure, the Turing degrees ($\mathcal{D}_T$). In effect, being transitive and reflexive $\leq_e$ itself induces an equivalence relation ($\equiv_e$) on the powerset of the natural numbers. As a result, two sets belong to the same equivalence class if they contain the same positive information content as stipulated by $\leq_e$. We call the structure of these equivalence classes, under the relation induced by $\leq_e$, the enumeration degrees ($\mathcal{D}_e$). This structure is an upper semi-lattice with zero degree ($\mathbf{0}_e$) corresponding to the class of Σ^0_1 sets. Moreover, there is a natural isomorphic embedding (ι) of $\mathcal{D}_T$ into $\mathcal{D}_e$. In other words $\mathcal{D}_e$ contains a copy of $\mathcal{D}_T$ as a substructure TOT , called the total degrees. Accordingly, $\mathcal{D}_e$ and its substructure TOT can be considered as a more general setting for the study of the Turing degrees. In the Turing degrees the jump operation is a function mapping any degree $\mathbf{a}$ to the degree of the relativized halting set of a/any set $A \in \mathbf{a}$. The jump of $\mathbf{a}$, written $\mathbf{a}'$ is strictly above $\mathbf{a}$ and displays specific *hardness* properties relative to $\mathbf{a}$. A jump operation for the enumeration degrees (with the same notation) was defined by Cooper [29]. This is defined in such a way that the jump is preserved under the embedding ι of $\mathcal{D}_T$ into $\mathcal{D}_e$. Thus $\mathcal{D}_T$ in fact embeds isomorphically into $\mathcal{D}_e$ complete with jump operation.

The global properties of the structure of the enumeration degrees have not yet been investigated thoroughly. Cooper [8] proved that unlike the Turing degrees the structure of the enumeration degrees does not have minimal elements. Nonetheless it follows from Calhoun and Slaman [6] that this structure is not dense. Case [7] proved that every countable ideal I of enumeration degrees has an exact pair, a pair of enumeration degrees $\mathbf{a}$ and $\mathbf{b}$, which are strictly above the elements of I, and such that the elements of I are exactly the ones that are bounded by both **a** and **b**. Rozinas [33] extended this result proving that this pair can always be chosen as a pair of total enumeration degrees. A set $\mathcal{B}$ of enumeration degrees is called an *automorphism base*, if every automorphism which preserves the elements of $\mathcal{B}$ is the identity. Sorbi [43] used the above mentioned results to prove the existence of two automorphism bases for the structure of the enumeration degrees - one containing exactly the total degrees and one containing the non-total enumeration degrees. Kalimullin [21] proved that the jump operator is first order definable in $\mathcal{D}_e$, giving one of the few first order definability results in the enumeration degrees. Slaman and Woodin [41] prove that the global theory of the enumeration degrees is computably isomorphic to second order arithmetic.

Far more work has been done in the local theory of the enumeration degrees. The enumeration jump gives rise to the local structure of the enumeration degrees $\mathcal{D}_e[\leq \mathbf{0}_e']$, consisting of all enumeration degrees reducible to $\mathbf{0}_e'$. This degree structure has a straightforward classification in terms of the Arithmetical Hierarchy as Cooper [9] proved that its elements are exactly the enumeration degrees of the Σ_2^0 sets. The preservation of the jump operator under the embedding ι shows that $\mathcal{D}_e[\leq \mathbf{0}_e']$ is itself an extension of the structure $\mathcal{D}_T[\leq \mathbf{0}']$. Furthermore the c.e. Turing degrees are also contained as a substructure of the $\mathcal{D}_e[\leq \mathbf{0}_e']$, namely they embed exactly onto the Π_1^0 enumeration degrees. The total enumeration degrees are a proper subclass of the Δ_2^0 enumeration degrees, as by Medvedev [30] there are Δ_2^0 quasi minimal enumeration degrees, degrees that do not bound any nonzero total enumeration degree.

This close connection between the Turing degrees and the enumeration degrees suggests that the study of the enumeration degrees can be useful for understanding the properties of $\mathcal{D}_T[\leq \mathbf{0}']$. Convincing proof of this argument is given by M. Soskova and Cooper [65], who obtain a strengthening of Harrington's non-splitting theorem for the Turing degrees as a straightforward corollary of a more general property of the enumeration degrees.

Working with the local structure of the enumeration degrees presents us with significant challenges. Compared to the *c.e.* Turing degrees which have nice approximation and are easier to work with, the Σ_2^0 enumeration degrees are far more complicated. Many new techniques for working in the local structure of the enumeration degrees have been developed. Cooper [9] proves that the Σ_2^0 enumeration degrees are dense and suggests the use of special approximations to Σ_2^0 sets, with sufficiently many *thin* stages. Nies and Sorbi [31] study the branching properties of the Σ_2^0 enumeration degrees and suggest the analogue for the *length of agreement* technique, a basic technique for c.e. constructions, for constructing $Sigma_2^0$ sets. Further techniques mainly connected with the priority method are developed by Cooper, Sorbi and Yi [34], by Soskova and Wu [66] and by Soskova [64], who study the cupping properties of $\mathcal{D}_e[\leq \mathbf{0}_e']$; by Arslanov and Sorbi [3] and by Soskova [62] who examine the relative splitting of $\mathbf{0}_e'$; by Cooper, Li, Sorbi and Yang [35] and by Soskova [63], who study the distribution of minimal pairs above $\mathbf{0}_e$. Techniques connected with the use of semi-recursive sets have been developed by Arslanov, Cooper and Kalimullin and used in [27] and [26].

The investigation of structural properties of $\mathcal{D}_e[\leq \mathbf{0}_e']$ has important applications to the study of the strength of its theory. In contrast to the Turing degrees, where much more is known, we have yet to discover the exact complexity of theory $\mathcal{D}_e[\leq$

$\mathbf{0}'_e]$. So far we know that by Slaman and Woodin [41] the theory of $\mathcal{D}_e[\leq \mathbf{0}'_e]$ is not decidable. Results towards disclosing the question of the decidability of fragments of the the first order theory of $\mathcal{D}_e[\leq \mathbf{0}'_e]$, such as the $\forall\exists$-theory, have also been made. Lempp and Sorbi [24] proved that every finite lattice can be embedded in $\mathcal{D}_e[\leq \mathbf{0}'_e]$ preserving least and greatest elements and Lempp, Slaman and Sorbi [36] solve the extension of embeddings problem, proving that it is decidable.

A further important characterization of the enumeration degrees is in terms of the properties of their jumps. The monotonicity properties of the jump operator allows us to define jump classes of enumeration degrees: an enumeration degree $\mathbf{a} \in \mathcal{D}_e[\leq \mathbf{0}'_e]$ is said to be low_n if $\mathbf{a}^{(n)}$ has the lowest possible value, namely $\mathbf{0}_e^{(n)}$ and dually a degree $\mathbf{a}$ is said to be $high_n$ if $\mathbf{a}^{(n)}$ has the highest possible value $0_e^{(n+1)}$. So far only the properties of the first levels of these jump classes have been investigated. Cooper and McEvoy [29] give a nice characterization of the low_1 enumeration degrees in terms of their approximation and in terms of the ideals they generate. They define the notion of a Σ^0_2-high enumeration degree, which is later proved to coincide with the notion of a high_1 enumeration degree by Shore and Sorbi [38], giving an analogue of Cooper's high permitting technique for the enumeration degrees. Structural properties of these classes have been investigated by Giorgi, Sorbi and Yue [25].

As a result of this investigation the structure of the Σ^0_2 enumeration degrees reveals itself as extremely complicated, having many of the same anomalies that were observed and attracted the interest of many renowned mathematicians to the field of the c.e Turing degrees. By Ahmad [2] the two structures are not elementary equivalent. The reason and extent of these connections is yet to be clarified.

4.1.2. *ω-enumeration degrees.* Uniform reducibility between sequences of sets is examined in the article [52], where this reducibility is characterized via regular enumerations. The structure $\mathcal{D}_\omega$ is defined on the basis of uniform reducibility and studied in the articles [47] and [48]. It is shown that the structure of the enumeration degrees $\mathcal{D}_e$ is a substructure of $\mathcal{D}_\omega$, the first substructure in a series of substructures $\{Ds_n\}_{n<\omega}$, where $\mathcal{D}_n$ is induced by uniform reducibility between sequences of sets with length n. A jump operator is defined in $\mathcal{D}_\omega$. The local structure $\mathcal{D}_\omega(\leq_\omega \mathbf{0}'_\omega)$ consisting of all ω-enumeration degrees reducible to the first jump $\mathbf{0}'_\omega$ of the least element $\mathbf{0}_\omega$ is also examined and it is shown that this structure is dense.

Structural properties of $\mathcal{D}_\omega$ are examined in [18]. In this article the author establishes the existence of a minimal pair $\mathbf{x}, \mathbf{y}$ above $\mathbf{a}$ for every degree $\mathbf{a}$, which has furthermore the property that the n-th jumps of $\mathbf{x}$ and $\mathbf{y}$ form a minimal pair above the n-th jump of $\mathbf{a}$. Later on Ganchev shows that in sharp contrast to the enumeration degrees, not every sequence of ω-enumeration degrees has an exact pair.

In [51] the authors examine the properties of the jump operation. They show that the jump operator in $\mathcal{D}_\omega$ extends the jump operator in $\mathcal{D}_e$ and prove a jump inversion theorem, namely that for every pair of degrees $\mathbf{a}$ and $\mathbf{b}$, such that $\mathbf{a}' \leq_\omega \mathbf{b}$ there exists a least degree $\mathbf{x}$ satisfying the following properties:

$$\mathbf{a} \leq_\omega \mathbf{x} \ \&\ \mathbf{x}' = \mathbf{b}.$$

This property of the jump operator distinguishes $\mathcal{D}_\omega$ from the classical structures of the enumeration degrees $\mathcal{D}_e$ and of the Turing degrees $\mathcal{D}_T$ where such least jump inverts do not exist.

The existence of least jump inverts in $\mathcal{D}_\omega$ an important ingredient for the proof of the first order definability of the enumeration degrees in the structure ${\mathcal{D}_\omega}'$ of the ω-enumeration degrees with jump, which is then used to prove that the automorphism groups of ${\mathcal{D}_\omega}'$ and $\mathcal{D}_e$ are isomorphic. This last result suggests a new approach towards the central problem in the theory of the enumeration degrees and the theory of the Turing degrees about the characterization of the automorphisms of these structures.

The benefit of studying the automorphism properties of $\mathcal{D}_\omega$ comes from the existence of more elements with new properties, unaccounted in Ds_e. Elements of this sort are examined in the second part of $\mathcal{D}_T$. For every natural number $n \geq 1$ let o_n denote the the leats degree, whose n-th jump is $\mathbf{0}_\omega^{(n+1)}$. We obtain a monotonously decreasing sequence of degrees below $\mathbf{0}'_\omega$:

$$\mathbf{0}_\omega \cdots <_\omega o_{n+1} <_\omega o_n <_\omega \ldots o_1 <_\omega \mathbf{0}'_\omega.$$

A degree $\mathbf{x}$ is called *almost zero* if $(\forall n)(\mathbf{x} \leq_\omega o_n)$. It is shown that the almost zero degrees form a nontrivial ideal and can be characterized as the degrees of sequences $\{A_n\}$, where $(\forall n)(A_n \leq_e \emptyset^{(n)}$.

The almost zero degrees turn out to be useful for characterizing the class H, consisting of the degrees below $\mathbf{0}'_\omega$ whose n-th jump is $\mathbf{0}_\omega^{(n+1)}$ for some natural number n, and the class L, consisting of the degrees below $\mathbf{0}'_\omega$ whose n-th jump is $\mathbf{0}_\omega^{(n)}$ for some natural number n. It is shown that the elements of H are exactly the ones that dominate every almost zero degree and the elements of L are the ones that do not dominate any nonzero almost zero degree.

The above result gives the first characterization of the classes H and L in $\mathcal{D}_e$. This follows from the fact that the jump operators in $\mathcal{D}_e$ and $\mathcal{D}_\omega$ agree.

The presented, previously obtained results are proof that the study of the global, as well as the local structure of the ω-enumeration degrees can turn out useful for the solution of significant problems in the classical structure of the enumeration degrees.

4.1.3. *Spectra of structures.* The effective content of mathematical theories and models begins with the works of Ash and Nerode [5] and attracts increasing interest lately. This topic is investigated in effective model theory. One approach to assessing the complexity of a given structure is its spectrum, a notion that was introduced by L. Richter in [32]. The spectrum of a structure is the set of the Turing degrees of the diagrams of all isomorphic copies of the structure. In classical model theory every two isomorphic structures are regarded as equivalent as they possess the same properties. With respect to the effective content of a structure and the computable functions in it two isomorphic structures can be completely different. In this way the introduced notion of spectrum is invariant with respect to the representation of a structure and gives simultaneously a measure for the complexity of a structure. L. Richter shows that if the spectrum of a linear order has a least element, called the degree of a structure, then it is $\mathbf{0}_T$. K. Jockusch suggest a more general notion for the complexity of a structure - the least element of the α-th-jump of elements of the spectrum for every computable ordinal α, called the

α-jump degree. J. Knight shows in [22], that a linear order has a 1-jump degree, namely $\mathbf{0}'_T$. Ash, Jockusch and Downey and J. Knight [4, 16] give examples of structures for every computable ordinal α, which have an α-jump degree and do not have a β-jump degree for every $\beta < \alpha$. Slaman [40] and Wehner [**?**] construct structures whose spectra consist of all nonzero Turing degrees.

In [46] Soskov initiates the study of the notion of a spectrum as a set of enumeration degrees, of all and not only the injective representations of a system. The advantage of this approach is that the spectra are closed upwards with respect to total enumeration degrees. He introduces the notion of a co-spectrum as the set of a lower bounds to the elements of the spectrum. He proves a series of properties of the spectra and their co-spectra, such as a minimal pair theorem and the existence of quasi-minimal degrees. For every enumeration degree **a**, Soskov shows that there is a group, a subgroup of the rational numbers with degree **a**. Soskov shows that every countable ideal of enumeration degrees is a co-spectrum of a structure. Kalimullin shows that for the introduced by Soskov notion of spectrum there is a structure whose spectrum consists of all nonzero enumeration degrees.

In [60, 56, 61] A. Soskova and I. Soskov show that every jump spectrum is a spectrum of a structure. They prove a jump inversion theorem for spectra of structures. The presented approach, a combination of Moschovakis extensions and Marker extension of a structure, allow us to reduce the complicated construction of structures with an n-jump degree and without k-jump degree for $k < n$, to a simple application of the jump inversion theorem for spectra.

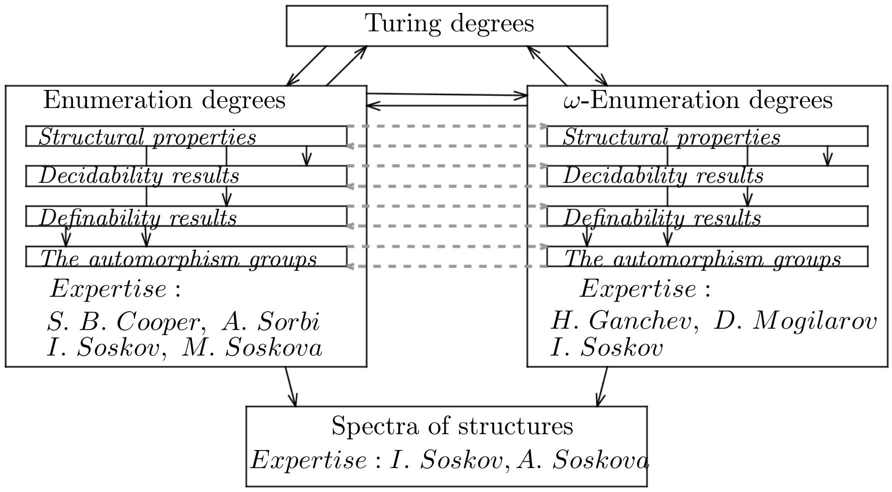
In [59, 53, 54, 57, 55] A. Soskova relativizes the notion of spectrum by examining the join and relative spectrum of a structure with respect to a finite number of given structures. She shows that all properties of the spectra and co-spectra are preserved for the relativized notions.

In [58] A. Soskova generalizes the notion of spectrum with respect to an infinite sequences of given sets withe respcet to the ω-enumeration degrees. The co-spectrum of an ω-spectrum is the set of ω-enumeration degrees, which are lower bounds of the ω-Spectrum. It turned out that not all properties of the spectra are true of the ω-spectra. Her graduate student St. Vatev gives a counterexample for some properties of the spectra, which are not true for the ω-spectra. Nevertheless A. Soskova shows that the basic properties, such as the existence of minimal pairs amd quasi-minimal degrees, are still realized. A characterization f the sets of ω-enumeration degrees which consist of the lower bounds of the k-jump spectrum of structures is given in terms of definable sets via Σ_k-computable formulas. The characterization of these sets as as the least ideal containing the k-th jumps of elements of the co-spectrum is of special interest.

These investigations are aimed at defining specific conditions under which an ideal of ω- enumeration degrees is a co-spectrum of structure, and at the more general question, when an upwards closed set of enumeration degrees is a spectrum of a structure.

4.2. **The scientific content of the project.** The main scientific problems which we propose are the following:

- Global and local structural properties of $\mathcal{D}_\omega$.
- Global and local decidability in $\mathcal{D}_\omega{}'$ and $\mathcal{D}_e$.
- Global and local definability in $\mathcal{D}_\omega{}'$ and $\mathcal{D}_e$.
- Characterization of the automorphisms of $\mathcal{D}_\omega{}'(\mathcal{D}_e)$.
- Investigation of the ω-spectra of abstract structures.



4.2.1. *Investigation of the global and local structural properties of $\mathcal{D}_\omega$.* The investigations of the global structural properties in $\mathcal{D}_\omega$ shall manly be concentrated mainly on the characterization of countable ideals which have an *exact pair*, a pair of degrees $\mathbf{a}, \mathbf{b}$ which are an upper bound to the elements of the ideal I and such that if $\mathbf{x} \leq_\omega \mathbf{a}$ and $\mathbf{x} \leq_\omega \mathbf{b}$ then $\mathbf{x} \in I$.

It turns out that if the ideal I has an exact pair $\mathbf{a}, \mathbf{b}$ then $\mathbf{a}', \mathbf{b}'$ is an exact pair for the ideal I', consisting of the jumps of elements in I. This fact yields that the ideal of the arithmetic ω-enumeration degrees, generated by the the jumps of $\mathbf{0}_\omega$ does not have an exact pair. On the other hand every co-spectrum of an abstract structure does have an exact pair and hence every prime ideal has such a pair. The characterization of the ideal which have an exact pair is considered as a key step in the solution of the problem to determine these sets of ω-enumeration degrees which are spectra or co-spectra of abstract structures.

Another natural problem connected with countable ideals is the investigation of their upper bounds. It is well known that in $\mathcal{D}_T$ every countable ideal has a least upper bound, in contrast to $\mathcal{D}_e$ where this is still an open question. In $\mathcal{D}_\omega$ this topic has not been studied extensively. The only result on this topic is published in [51], and states that the ideal of the almost zero degrees below $\mathbf{0}'_\omega$ does not have a least upper bound below $\mathbf{0}'_\omega$.

The established density of the ω-degrees below $\mathbf{0}'_\omega$, [48], leads to the natural question about the density of the whole structure. It is known that $\mathcal{D}_e$ is not dense, which suggests the possibility that $\mathcal{D}_\omega$ is also not dense. The investigation of this problem will shed light on the connections between the two structures.

The definition of the almost zero degrees, [51], and their role in the characterization of the classes H and L gives rise to the question about the distribution of these degrees in the whole structures $\mathcal{D}_\omega$ and about the investigation of their properties. There are many open problems related to this topic: Are there incomparable almost zero degrees? Is there a minimal pair above $\mathbf{0}_\omega$ of almost zero degrees? What are the properties of degrees which dominate certain classes of almost zero degrees?

Problems that will be investigated, regarding the local structure of $\mathcal{D}_\omega$, include the distribution of minimal pairs below $\mathbf{0}'_\omega$ as well as the distribution of relatives splittings of $\mathbf{0}'_\omega$. These topics have been thoroughly investigated in $\mathcal{D}_e$, [29, 63]. In $\mathcal{D}_\omega$ these are open questions.

4.2.2. *Global and local first order definability in* ${\mathcal{D}_\omega}'$ *and* $\mathcal{D}_e$. It follows from the results in [51] that the structures $\mathcal{D}_e$ is first order definable in ${\mathcal{D}_\omega}'$, and hence every set of enumeration degrees which is definable in $\mathcal{D}_e$ is also definable in ${\mathcal{D}_\omega}'$. An important question is whether the reverse is true. We expect to find that the answer to this question is negative, proving the hypothesis that ${\mathcal{D}_\omega}'$ is richer in structure than $\mathcal{D}_e$.

An important step towards global definability in $\mathcal{D}_\omega$ is the generalization of Slaman and Woodin's coding lemma, [42, 41], for $\mathcal{D}_\omega$. This generalization would allow us to find a first order formula, which using parameters defines every countable set of ω-enumeration degrees. In this way we would be able to quantify over arbitrary countable relations and then to obtain definitions of the arithmetic ω-enumeration degrees, of the almost zero enumeration degrees, the classes L and H, etc.

The main problem related to the local definability in the structure $\mathcal{D}_\omega[\leq_\omega \mathbf{0}'_\omega]$ will be finding a definition of o_n, $n = 1, 2, \ldots$ and of the set $\mathcal{O} = \{o_n : n \geq 1\}$. This problem is closely related to certain structural properties of the enumeration degrees below $\mathbf{0}'_e$, such as the existence of minimal pairs of degrees with special properties or the existence of certain relative splittings of $\mathbf{0}'_e$.

After establishing the above mentioned local properties of the enumeration degrees we would be able to locally define in $\mathcal{D}_\omega$ the enumeration degrees below $\mathbf{0}'_e$, the classes H and L, etc.

The investigation of the relations between the local theories of the enumeration and ω-enumeration degrees sets the question about the definability in the local structure of the enumeration degrees. Open question concern the definability of certain classes with a natural description such as L, H, the quasi-minimal degrees,the total degrees, etc.

The study of global and local definability is closely connected to the study of the automorphisms of the global and local structures of the enumeration and ω-enumeration degrees.

4.2.3. *Characterizing the automorphisms of* ${\mathcal{D}_\omega}'(\mathcal{D}_e)$*.* As was noted previously, it is proved in [51] that the automorphism groups of ${\mathcal{D}_\omega}'$ and $\mathcal{D}_e$ are isomorphic. As a side effect of this proof we get that the automorphisms of ${\mathcal{D}_\omega}'$ are extensions of the automorphisms in $\mathcal{D}_e$ and that if two automorphisms of ${\mathcal{D}_\omega}'$ agree on $\mathcal{D}_e$ then they agree everywhere. Another result from this article shows that every automorphism of ${\mathcal{D}_\omega}'(\mathcal{D}_e)$ preserves every element above the fourth jump of $\mathbf{0}$. In ${\mathcal{D}_\omega}'$ every automorphism preserves also every element below o_4.

Recall that a set B of enumeration degrees is called an *automorphism base*, if every automorphism which preserves the elements of B is the identity. It follows from the result described above that every automorphism base for $\mathcal{D}_e$ is an automorphism base for $\mathcal{D}_\omega$. An important question is wether or not one can find an automorphism base of $\mathcal{D}_\omega{}'$, which does not contain elements from $\mathcal{D}_e$. This question can be solved if one finds an isomorphic embedding of $\mathcal{D}_e$ which does not contain enumeration degrees. In this way the question of the characterization of the automorphism bases is reduced to investigating the possible embeddings of $\mathcal{D}_e$ in $\mathcal{D}_\omega{}'$. Finding new automorphism bases in $\mathcal{D}_\omega{}'$ would shed light onto problems connected with first order definability in $\mathcal{D}_e$.

A characterization of the automorphism bases in the local substructures of $\mathcal{D}_\omega$ and $\mathcal{D}_e$ has nod been found. An open problem is also to determine the connection between the automorphisms of these structures. One possible step towards the clarification of this connection could be related to the definability of the enumeration degrees below $\mathbf{0}_e'$ in the local substructure of $\mathcal{D}_\omega$, which was discussed above.

4.2.4. *Studying the ω-spectra of abstract structures.* The spectra and co-spectra of structures are sets of ω-enumeration degrees with certain specific properties. The problem related to finding a characterization of the countable ideals of ω-enumeration degrees which are co-spectra of structures is closely connected to the problems we set in 4.2.1.

Another goal is the generalization of the jump inversion theorem from [60] for the ω-spectra and the study of its corollaries.

We shall also investigate the definability of the ω-spectra via first order formulae. This question connects with the topic of the stability of ω-spectra under the automorphisms of $\mathcal{D}_\omega$.

4.3. **Description of the used scientific methodology.** Our intention is to develop the existing techniques, used in the Turing degrees and the enumeration degrees, by adapting them for the ω-enumeration degrees:

- The forcing method and regular enumeration.
- Good approximations to sequences of sets uniformly reducible to $\{\emptyset^{(n)}\}_{n<\omega}$.
- The priority method.
- Embedding finite lattices and semi-lattices.
- Embedding partial orders.
- Marker extensions.

4.3.1. *The forcing method and regular enumeration.* The forcing method is widely used for investigating the global structural properties of both the Turing degrees and the enumeration degrees. In the works of [50, 52] the authors construct generic objects, called regular enumerations, which code in their jumps a given sequence of sets. This techniques is used for the characterization of the uniform reducibility. The forcing method is used to prove the existence of minimal pairs of enumeration degrees below every ω-enumeration degree.

The generic enumeration degrees have been studied in the works of Copestake [10]. M. Soskova has also used genericity to prove certain structural properties in the local structure of the enumeration degrees in [63] and jointly with G. Wu in [66]. The exact role of genericity in the enumeration degrees s not completely clear yet and demands further investigation. Their importance is however indisputable and suggests that it would be useful to find the right analogous notion of a generic sequence of sets.

4.3.2. *Good approximations to sequences of sets reducible to $\emptyset^{(n)}_{n<\omega}$.* Good approximations to sequences of sets are introduced in [48]and are based on good approximations to Σ^0_2 sets [8, 9, 23].

Good approximations are then used in the article [48] in the proof of the density of the local structure of the ω-enumeration degrees and in [51] in the characterization of the low and high degrees.

The use of good approximations is especially important for investigating the properties of the local structure if the enumeration degrees and the ω-enumeration degrees. It plays a main role also in the adaptation of the priority method for the ω-enumeration degrees.

4.3.3. *The priority method.* The priority method originates from the renown works of Friedberg and of Muchnik on Post's problem in the 50's. This method is further developed and considered a powerful tool for investigating the degrees of objects which can be approximated. These are for example the computably enumerable sets and their Turing degrees, Σ^0_2 sets end their enumeration degrees, etc.

The contemporary development of this method can be seen in the works, connected with the local theory of the enumeration degrees [8, 9, 34, 25, 63, 62, 65, 64, 66].

The priority method in the ω-enumeration degrees is used in the article [48] in the proof of the density of the local structure of theω-enumeration degrees degrees and in [51] in the characterization of the low and high degrees.

The development of the priority method for the ω-enumeration degrees is one of the important goals of this project.

4.3.4. *Marker extensions.* Marker extensions are introduced in [28]. They are a model-theoretic constructions and are originally used precisely in model theory. The computable content of this construction is established in the work of Goncharov and Khoussainov [20]. Soskov and A. Soskova [60] use Marker extensions in the proof of the jump inversion theorem for spectra of structures. The adaptation of Marker extensions for the ω-spectra is expected to be a powerful tool for investigating their properties.

4.4. **Expected impact and results.** The goal of this project is to conduct parallel research on the structures of the enumeration degrees and the ω- enumeration degrees. This research is meant to facilitate the characterization of the definability in these structures and the automorphism groups of both structures.

We expect to obtain substantial mathematical contributions in the development of the used methods and techniques, as well as in concrete problems connected with the goal that is set out.

The importance of the envisioned problems and the innovative approach to their solution justify the expectation that the obtained results will have a substantial impact on the whole development of the field.

Joint work with the best European specialists in the field, S. B. Cooper and A. Sorbi gives a good base for the development of the Bulgarian school in computability theory.

The participation of the young bulgarian researchers, M. Soskova, H. Ganchev and D. Mogilarov will contribute to their development as specialists in the field of computability theory.

4.5. **Favorable environment for young researchers.** Three of the members included in this project are young researches. Two more positions for young researchers are planned - for a postgraduate student and for a research collaborator. In addition to the scientific research opportunities, young researchers will have the following opportunities:

- A supplement to their salary.
- Yearly specialization of up to 20 days in the participating research centers in Leeds and Seina.
- Yearly participation in the conference series "Logic Colloquium" and "Computability in Europe".

4.5.1. *Salary supplement.* From the total amount of 127 507 intended for supplementary payments, 92 372 will be distributed to young researchers. From these 64 600 will be used to attract one research collaborator and two postgraduate students.

4.5.2. *Yearly specialization of up to 20 days in the participating research centers.* The project includes short-term specializations of up to 20 days for the young participants in the University of Leeds and the University of Siena, three each year. These short-term specializations will give the opportunity to the young scientists to come in direct contact with the foreign researchers participating in the project.

4.5.3. *Yearly participation in the conference series "Logic Colloquium" and "Computability in Europe".* There are two main scientific conferences in the field, which are held every year. These are "Computability in Europe"(CiE) and "Logic Colloquium"(LC).

The conference series LC is initiated more than 40 years. At the moment LC is considered as the most important event in the field of Logic of the year. The conference is organized by the Association for Symbolic Logic (ASL) with a program committee of internationally acclaimed scientists. In 2009 LC will be held in Sofia.

The conference series CiE starts in 2005. Past venues of this conference are Amsterdam, Swansea, Siena and Athens. In 2009 CiE will be held in Heidelberg, in 2010 it will be held in the Azores islands, Portugal and in 2011 it will be held in Sofia. The conferences are intended to create a bridge between computability theory and computer science. They attract many scientists from both fields. Members of the program committee are internationally acclaimed scientists.

4.6. **Potential for the future development of the research group.** The suggested investigations are of fundamental character and will certainly not finish with the end of this project.

In the event of a successful end to this project the group will have reinforced potential for participation in further projects, funded by both bulgarian and international science funds.

The group is centered at the department of mathematical logic and its applications, Faculty for mathematics and informatics at Sofia University. Young participants will be attracted into the work of the department and shall carry on the traditions of the school in computability in Bulgaria.

4.7. **Potential for the transfer of knowledge and applications.** The dissemination of the results will be realized via contributed talks at conferences and publications of articles in scientific journals. The resources provided by the internet shall be used to achieve a wider dissemination and clarifications of the obtained results.

A main goal is to influence the growing society of mathematicians, computer scientists and theoretical physicists, working on questions concerning computability and incomputability in nature and centered around the conference series "Computability in Europe". This is precisely where we expect to discover possible practical applications of the obtained results.

5. Future development of the project

After the completion of the project the staff will continue to work into two main directions :

- Continuation of the research in the area.
- Participation of the young scientists in other international projects.

5.1. **Continuation of the research in the area.** In case of successful completion of the project the group will face a lot of new challenges. On the first p[lace this is the development of the theory of the ω-enumeration degrees on the basis of the Turing reducibility. Another important problem is the application of the obtained results in the Effective Model Theory and Abstract Data Types which will relate our research to more practical areas of the Computability Theory and of the Computer Science.

5.2. **Participation of the young scientists in other international projects.** The research, the participation in conferences and the specializations abroad shall give a opportunity for the young scientists to integrate in the international scientific environment in Europe. Such an integration is of great importance for their academic development.

6. Dissemination of the results

The planned result dissemination actions aim to give broad popularity of the carried out scientific research and to enlarge the skills of the research group working on the project. One of the important tasks of the project is disseminating the results. The main means for this purpose are publishing papers in renowned periodical mathematical journals and giving talks during conferences and colloquiums. Another way of dissimination is writing PhD thesis, which is the main task of the PhD students involved. Another one is the creation of an Internet page containing the main results of the project, as papers, announcements for conferences and links to the major sources of information about global and local theory of the enumeration degrees, the omega-enumeration degrees and spectra of structures. The proceedings of the Computability in Europe (CiE) and Logic Colloquium (LC) meetings taking place in Bulgaria will also help spreading the results of the project.

The main means of result spreading may be gathered in the following groups:

- Results publishing
- Conference participation
- Organizing conferences
- PhD theses
- Internet page

6.1. **Publishing of the results.** The most important task of the the project members is publishing their research results in renowned international periodic journals as the Journal of Symbolic Logic, Annals of Pure and Applied Logic, Journal of Logic and Computation,Lecture Notes in Logic, Lecture Notes in Comp. Sci, Bulletin in Symbolic Logic. The last two are traditional for the conferences CiE and LC. A large number of papers have been already published in this journals by the project members.

6.2. **Conference participation.** The project participants are members of the CiE organization. CiE organization organizes annually a conference covering all the project topics. As a rule all project members take part in this conference either as organizers or members of the programme committee, or with contributed talks during the conference. S.B. Cooper and A. Sorbi are one of the founding members of this organization. I. Soskov is a programme committee member. A. Soskova, H. Ganchev and M. Soskova participate with contributed talks.

The Logic Colloquium is another major conference that may be attained by project members. It takes place annually during summer time in Europe. Giving talks in LC shall give an opportunity to the project members to announce their results to the logicians around the world.

The Panhellenic Logic Symposium is a meeting in Greece, where logicians from Balkan countries present their results. As a tradition project members are among the organizers and participants in these meetings.

6.3. **Organizing Conferences.** Publishing materials from the project meetings is planned.

The Logic Colloquium in 2009 will take place in Bulgaria. I. Soskov is a member of the programme committee and A. Soskova is chair of the organizing committee. M. Soskova is a member of the organizing committee. This big world forum shall help the dissipation of the ideas of the project.

The CiE meeting in 2011 is anticipated to take place in Bulgaria.

There are some ideas about enlarging the Panhellenic Logic Symposium to a real Balkan conference to be held in 2011 in Bulgaria.

The results presented during these conferences shall be published in different mathematical logic journals.

6.4. **PhD Theses.** PhD students M. Soskova and H. Ganchev have already finished their theses. D. Mogilarov is still in process of writing it. A new PhD position is planned within the project. The results obtained during the PhD training shall be presented to the collective and shall be published in renowned journals.

6.5. **Internet page.** A project Internet page is planned. Its scopes are:

- popularizing of the project main ideas;
- giving access to preprints of the research results;
- announcing conferences
- giving easy access to the major sources of information about global and local theory of the enumeration degrees, the omega-enumeration degrees and spectra of structures.

A report for the accomplished work is also planned.

7. Management of the project

The coordinator will take the main responsibility for the organization of the project. During the visits of the foreign participants in Bulgaria the obtained results will be discussed and concrete plans for further research will be designed.

The project will use the administrative base and accountancy of the Faculty of mathematics and informatics at Sofia University.

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